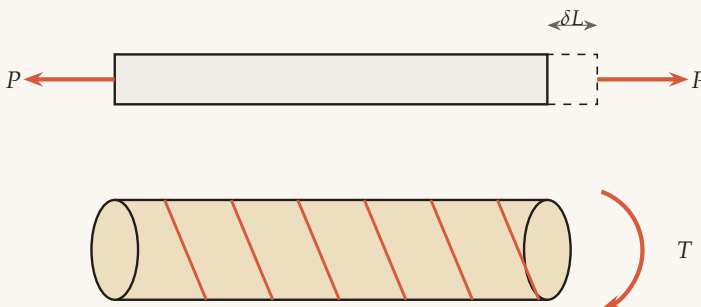


STRENGTH OF MATERIALS

Formula-to-Solution Starter

**Understand the essentials. Follow the method.
Solve your first problems.**

Direct stress and strain • Poisson's ratio • Torsion of shafts
4 worked examples • 5-question self-test with full answers



Start here

This guide is for you if formulas in Strength of Materials look like a wall of symbols, if you can follow your lecturer's example but freeze when the numbers change, or if you are not sure which formula a question wants.

What you will be able to do after one or two study sessions:

- say what stress, strain, Young's modulus and Poisson's ratio mean, with units;
- find the stress, extension and change in diameter of a loaded bar;
- find the torque in a shaft from its power and speed;
- find the shear stress and twist in a shaft, and choose a shaft diameter;
- check your own answer before you hand it in.

Why these topics

We read the Strength of Materials section of 12 Module II papers (8 dated Oct/Nov 2019 to Mar/Apr 2025, plus 4 undated). **Torsion of shafts** appeared in all 12. **Stress, strain and the elastic constants** appeared in 9. Both rest on the same idea: a load causes stress, stress causes strain, and a material constant links them. Learn that idea once and you can use it in both topics. These counts describe past papers only; they do not predict the next paper.

What this guide does not cover

This is a focused starter, not the whole syllabus. It leaves out shear force and bending moment, bending stress, beam deflection, thin cylinders, strain energy and springs. Those are taught in the complete SomaReady Strength of Materials guide (see the last page).

How to use it

1. Read each idea and its picture before the formula.
2. In each worked example, cover the working and try the next step yourself first.
3. Do the self-test on page 9 without looking back. Then mark it with the worked answers.
4. Keep the quick-reference page (page 11) for revision.

SYMBOL WATCH

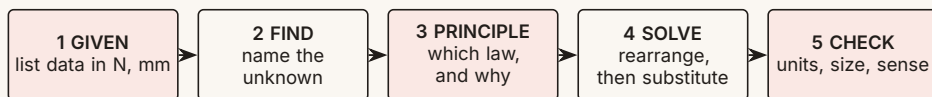
T means **torque** (N mm or N m) in this guide. d is a **diameter** and r , R are **radii**, all in mm. Greek letters: σ (sigma) normal stress, ϵ (epsilon) normal strain, ν (nu) Poisson's ratio, τ (tau) shear stress, γ (gamma) shear strain, θ (theta) angle of twist.

PART 1

From formula to solution

Most marks are lost before any arithmetic starts: wrong units, the wrong formula, or a rearrangement done in a rush. A fixed routine prevents all three.

1.1 The five-question method



Every worked example in this guide follows these five questions, in this order:

- 1. What is given?** Write each quantity with its symbol and unit. Convert to newtons and millimetres now, not later.
- 2. What is asked?** Name the unknown and its unit before you look for a formula.
- 3. Which principle applies, and why?** Say it in words (“the bar carries a direct load, so $\sigma = P/A$ ”). Examiners give marks for this.
- 4. Rearrange, then substitute.** Make the unknown the subject using symbols first. Then put in numbers, with units.
- 5. Is the answer reasonable?** Check the unit, the size and the sense (tension or compression, bigger or smaller).

1.2 Units: work in N and mm

If forces are in newtons (N) and lengths in millimetres (mm), stresses come out directly in N/mm^2 :

$$1 \text{ N/mm}^2 = 1 \text{ MN/m}^2 = 1 \text{ MPa}, \quad 1 \text{ GN/m}^2 = 1000 \text{ N/mm}^2$$

So $E = 200 \text{ GN/m}^2$ is $200\,000 \text{ N/mm}^2$; 50 kN is $50\,000 \text{ N}$; 1.5 m is 1500 mm ; and a torque of 2 kN m is $2 \times 10^6 \text{ N mm}$.

! COMMON MISTAKE • UNITS

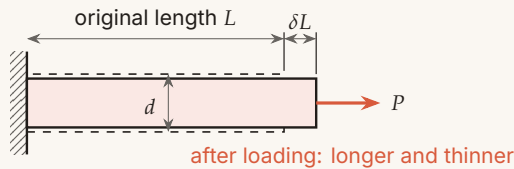
- Mixing kN with N, or m with mm, inside one formula. Convert everything first.
- Using the diameter where the formula wants a radius. Write “ $d = \dots$, so $r = \dots$ ” at the start.

PART 2

Direct stress and strain

Pull on a steel rod and it stretches a tiny amount. Stress tells you how hard the material is working; strain tells you how much it has deformed.

2.1 Stress, strain and Young's modulus



DEFINITION • STRESS AND STRAIN

Stress is the internal force carried per unit area of the cross-section, $\sigma = P/A$. It is *tensile* when the load pulls and *compressive* when it pushes. **Strain** is the change in length per unit of original length, $\epsilon = \delta L/L$. Strain is a ratio of two lengths, so it has **no unit**.

What links them. Up to the limit of proportionality, stress is proportional to strain (Hooke’s law). The constant is **Young’s modulus** E : the stress needed per unit strain. A stiff material (steel) has a large E ; a flexible one (timber) has a small E .

KEY FORMULA • DIRECT STRESS, STRAIN AND EXTENSION

$$\sigma = \frac{P}{A}, \quad \epsilon = \frac{\delta L}{L}, \quad E = \frac{\sigma}{\epsilon} \implies \delta L = \frac{PL}{AE}$$

Symbol	Meaning	SI unit
P	axial load (along the bar’s axis)	N
A	cross-sectional area; for a round bar $A = \pi d^2/4$	mm ²
σ	direct (normal) stress	N/mm ²
$L; \delta L$	original length; change in length	mm
ϵ	direct strain (no unit)	–
E	Young’s modulus (steel $\approx 200 \text{ GN/m}^2 = 200\,000 \text{ N/mm}^2$)	N/mm ²

When it applies. The load acts along the axis, the bar has a uniform section, and the stress stays below the limit of proportionality. **How the last form comes out:** $E = \sigma/\epsilon = (P/A)/(\delta L/L)$; multiply both sides by δL and divide by E to get $\delta L = PL/AE$.

2.2 Poisson's ratio

A bar that gets longer also gets *thinner* (see the figure). The **lateral strain** is the change in diameter divided by the diameter, $\delta d/d$. For a given material it is a fixed fraction of the

longitudinal strain:

KEY FORMULA • POISSON'S RATIO

$$\nu = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\delta d/d}{\delta L/L} \implies \delta d = \nu \varepsilon d$$

ν has no unit. For metals it is about 0.25–0.35, and it can never exceed 0.5. A value above 0.5 in your working means a slip. The two strains always have *opposite* senses: longer means thinner; shorter means fatter.

WORKED EXAMPLE E1

DIRECT APPLICATION

A steel rod 20 mm in diameter and 1.5 m long carries a tensile load of 50 kN. $E = 200 \text{ GN/m}^2$ and $\nu = 0.3$. Find the stress, the extension and the change in diameter.

1. GIVEN (IN N AND MM)

$P = 50\,000 \text{ N}$; $d = 20 \text{ mm}$; $L = 1500 \text{ mm}$; $E = 200\,000 \text{ N/mm}^2$; $\nu = 0.3$.

2. FIND

$\sigma \text{ (N/mm}^2\text{)}, \delta L \text{ (mm)} \text{ and } \delta d \text{ (mm)}.$

3. PRINCIPLE

The load acts along the rod's axis, so the direct-stress formulas apply. Poisson's ratio links the change in diameter to the strain.

4. SOLVE

$A = \pi(20)^2/4 = 314.2 \text{ mm}^2$, so $\sigma = 50\,000/314.2 = 159.2 \text{ N/mm}^2$ (tension).

$\varepsilon = \sigma/E = 159.2/200\,000 = 7.958 \times 10^{-4}$; $\delta L = \varepsilon L = 7.958 \times 10^{-4} \times 1500 = 1.194 \text{ mm}.$

$\delta d = \nu \varepsilon d = 0.3 \times 7.958 \times 10^{-4} \times 20 = 0.00477 \text{ mm}$ (a decrease).

$\sigma = 159 \text{ N/mm}^2$; $\delta L = 1.19 \text{ mm longer}$; $\delta d = 0.0048 \text{ mm thinner}$

5. CHECK

Second route: $\delta L = PL/AE = 50\,000 \times 1500/(314.2 \times 200\,000) = 1.194 \text{ mm}$. A steel rod stretching about 1 mm over 1.5 m is realistic, and the diameter change is far smaller, as it should be.

WORKED EXAMPLE E2

REARRANGING: CHOOSE A DIAMETER

A round steel hanger 2 m long must carry 80 kN. The working stress must not exceed 120 N/mm^2 ; $E = 200 \text{ GN/m}^2$. Find the least diameter, choose a practical size in whole millimetres, and find the extension with that size.

1. GIVEN

$P = 80\,000 \text{ N}$; $\sigma_{\text{allow}} = 120 \text{ N/mm}^2$; $L = 2000 \text{ mm}$; $E = 200\,000 \text{ N/mm}^2$.

2. FIND

The least diameter $d \text{ (mm)}$; then $\delta L \text{ (mm)}$ for the chosen size.

3. PRINCIPLE

The stress must not exceed the allowed value, so the area must be at least P/σ_{allow} .

continued on next page →

WORKED EXAMPLE E2 – CONTINUED

REARRANGING: CHOOSE A DIAMETER

4. REARRANGE FIRST, THEN SUBSTITUTE

$$\sigma = P/A \Rightarrow A = P/\sigma = 80\,000/120 = 666.7 \text{ mm}^2.$$

$$A = \pi d^2/4 \Rightarrow d = \sqrt{4A/\pi} = \sqrt{4 \times 666.7/\pi} = 29.13 \text{ mm. Round up: use } d = 30 \text{ mm.}$$

$$\text{With } d = 30 \text{ mm: } A = 706.9 \text{ mm}^2, \sigma = 113.2 \text{ N/mm}^2, \text{ and } \delta L = \frac{80\,000 \times 2000}{706.9 \times 200\,000} = 1.132 \text{ mm.}$$

least $d = 29.1 \text{ mm}$, so use 30 mm ; $\delta L = 1.13 \text{ mm}$

5. CHECK

Rounding up lowers the stress (113 is below 120), so the choice is safe. Rounding down to 29 mm would give 121 N/mm^2 : too high.

! COMMON MISTAKE • DIRECT STRESS

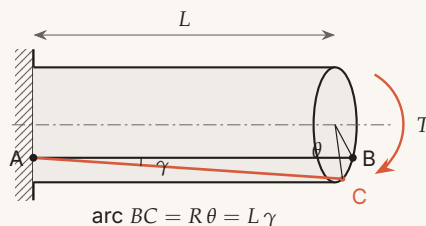
- Leaving E in GN/m^2 while using N and mm : the extension comes out 1000 times too small.
- Rounding a design diameter down. Always round up to the next practical size.
- Giving δd as an increase in a tension problem.

PART 3

Torsion of shafts

A shaft carries power from a motor to a machine by turning under torque. The torque twists the shaft and sets up shear stress in it. A good shaft is strong enough not to fail and stiff enough not to twist too much.

3.1 What happens when a shaft twists



Fix one end of a shaft and apply a torque T at the other. A straight line AB drawn along the surface moves to AC . The free end turns through the **angle of twist** θ , and the line tilts by the **shear strain** γ . The same arc gives $BC = R\theta = L\gamma$.

Shear stress and shear strain are linked by the **modulus of rigidity** G , in the same way that E links direct stress and strain: $\tau = G\gamma$.

3.2 The torsion equation

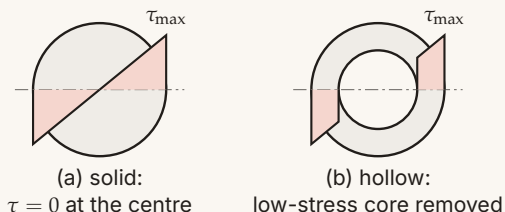
KEY FORMULA • TORSION OF A CIRCULAR SHAFT

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}, \quad P = \frac{2\pi NT}{60}$$

$$\text{Solid: } J = \frac{\pi d^4}{32}, \quad \tau_{\max} = \frac{16T}{\pi d^3} \quad \text{Hollow: } J = \frac{\pi(D^4 - d^4)}{32}$$

Symbol	Meaning	SI unit
T	torque carried by the shaft	N mm
J	polar second moment of area (how the area is spread about the axis)	mm ⁴
$\tau; r$	shear stress at radius r ; it is greatest at the surface, $r = R = d/2$	N/mm ² ; mm
G	modulus of rigidity (steel $\approx 80 \text{ GN/m}^2 = 80\,000 \text{ N/mm}^2$)	N/mm ²
$\theta; L$	angle of twist in radians ; length of shaft	rad; mm
$P; N$	power transmitted; shaft speed	W; rev/min

Reading the equation. Each pair of terms is a separate tool. Use $T/J = \tau/r$ for *strength* (stress), and $T/J = G\theta/L$ for *stiffness* (twist). The middle pair, $\tau/r = G\theta/L$, says the stress rises in a straight line from zero at the centre to its maximum at the surface:



That is why hollow shafts are efficient: the core they remove was only lightly stressed.

Assumptions. The material obeys Hooke's law and is the same throughout; the shaft is straight, circular and uniform; plane cross-sections stay plane and radii stay straight.

Where $\tau_{\max} = 16T/\pi d^3$ comes from: put $r = d/2$ and $J = \pi d^4/32$ into $\tau = Tr/J$.

WORKED EXAMPLE E3

DIRECT: STRESS AND TWIST

A solid steel shaft 50 mm in diameter transmits 40 kW at 600 rev/min. $G = 80 \text{ GN/m}^2$. Find the torque, the greatest shear stress and the angle of twist per metre length.

1. GIVEN

$P = 40\,000 \text{ W}$; $N = 600 \text{ rev/min}$; $d = 50 \text{ mm}$, so $R = 25 \text{ mm}$; $G = 80\,000 \text{ N/mm}^2$.

2. FIND

T (N m), $\tau_{\max}$ (N/mm²) and θ per metre (degrees).

3. PRINCIPLE

Power and speed give the torque. Then the torsion equation gives the stress (strength pair) and the twist (stiffness pair).

continued on next page →

WORKED EXAMPLE E3 – CONTINUED

DIRECT: STRESS AND TWIST

4. SOLVE

$$T = \frac{60P}{2\pi N} = \frac{60 \times 40\,000}{2\pi \times 600} = 636.6 \text{ N m} = 636\,620 \text{ N mm.}$$

$$J = \pi(50)^4/32 = 613\,592 \text{ mm}^4; \tau_{\max} = \frac{TR}{J} = \frac{636\,620 \times 25}{613\,592} = 25.94 \text{ N/mm}^2.$$

$$\text{Twist over } L = 1000 \text{ mm: } \theta = \frac{TL}{GJ} = \frac{636\,620 \times 1000}{80\,000 \times 613\,592} = 0.01297 \text{ rad} = 0.743^\circ.$$

$$T = 637 \text{ N m; } \tau_{\max} = 25.9 \text{ N/mm}^2; 0.743^\circ \text{ per metre}$$

5. CHECK

Second route for the stress: $16T/\pi d^3 = 16 \times 636\,620/(\pi \times 125\,000) = 25.94$. Radians to degrees: multiply by $180/\pi$. Less than 1° per metre is a typical limit, so this shaft is stiff enough for most uses.

WORKED EXAMPLE E4

EXAM STYLE: DESIGN FOR STRENGTH AND STIFFNESS

A solid steel shaft must transmit 25 kW at 300 rev/min. The shear stress must not exceed 60 N/mm^2 , and the twist must not exceed 1° over a length of 2 m. $G = 80 \text{ GN/m}^2$. Find the least diameter.

1. GIVEN

$P = 25\,000 \text{ W}$; $N = 300 \text{ rev/min}$; $\tau_{\text{allow}} = 60 \text{ N/mm}^2$; $\theta = 1^\circ = \pi/180 = 0.01745 \text{ rad}$ over $L = 2000 \text{ mm}$.

2. FIND

The least diameter d (mm) that satisfies *both* limits.

3. PRINCIPLE

Two separate conditions: the strength pair $T/J = \tau/r$ and the stiffness pair $T/J = G\theta/L$. Find d from each; the larger one meets both.

4. SOLVE

$$T = \frac{60 \times 25\,000}{2\pi \times 300} = 795.8 \text{ N m} = 795\,775 \text{ N mm.}$$

$$\text{Strength: } \tau = 16T/\pi d^3 \Rightarrow d^3 = \frac{16T}{\pi \tau} = \frac{16 \times 795\,775}{\pi \times 60} = 67\,547, \text{ so } d = 40.7 \text{ mm.}$$

$$\text{Stiffness: } J = \frac{TL}{G\theta} = \frac{795\,775 \times 2000}{80\,000 \times 0.01745} = 1.140 \times 10^6 \text{ mm}^4;$$

$$d^4 = \frac{32J}{\pi} = 1.161 \times 10^7, \text{ so } d = 58.4 \text{ mm.}$$

least diameter 58.4 mm (stiffness governs); use a 60 mm shaft

5. CHECK

At 58.4 mm the stress is only $16T/\pi d^3 = 20.4 \text{ N/mm}^2$, well under 60. The twist limit, not the stress limit, decides the size. Always work out both.

! COMMON MISTAKE • TORSION

- Putting θ in degrees into $G\theta/L$. The formula needs radians: $1^\circ = \pi/180 \text{ rad}$.
- Writing $P = T \times N$ with N in rev/min. Use $P = 2\pi NT/60$.
- Using $\pi d^4/64$ (the bending value) instead of $J = \pi d^4/32$.
- Stopping after the strength check when a twist limit is also given.

PART 4

Self-test

Five original practice questions, from warm-up to exam style. Use the five-question method on each. Allow about 45 minutes, then mark your work with the answers that follow.

PRACTICE • WORKED ANSWERS FROM PAGE 10

T1 WARM-UP (a) Give the SI unit of stress, strain, Young's modulus and Poisson's ratio. Say in one sentence why strain has no unit. (b) Convert $E = 210 \text{ GN/m}^2$ to N/mm^2 , and a torque of 3.5 kN m to N mm .

T2 DIRECT, COMPRESSION A short steel column 40 mm in diameter and 800 mm long carries a compressive load of 120 kN . $E = 210 \text{ GN/m}^2$, $\nu = 0.28$. Find the stress, the shortening and the change in diameter. Say whether the diameter grows or shrinks.

T3 FIND A CONSTANT A steel bar 25 mm in diameter carries a tensile load of 60 kN . $E = 200 \text{ GN/m}^2$. Its diameter is measured to decrease by 0.0046 mm . Find Poisson's ratio, and say whether the value is believable.

T4 HOLLOW SHAFT A hollow shaft has outside and inside diameters of 80 mm and 50 mm . The greatest shear stress allowed is 50 N/mm^2 . Find the greatest torque it can carry, and the power it can transmit at 400 rev/min .

T5 CHALLENGE Two shafts of the same material both have an outside diameter of 60 mm . One is solid; the other is hollow with a 40 mm bore. At the same greatest shear stress, find (a) the ratio of the torque the hollow shaft can carry to that of the solid shaft, (b) the ratio of their weights, (c) which shaft carries more torque per kilogram, and by how much.

Worked answers

If your method is different but correct, your answer will agree to about 3 significant figures. Look at *why* each step is taken, not only the number.

T1 (a) Stress: N/mm^2 (or N/m^2 , Pa). Strain: **no unit**. E : N/mm^2 , the same as stress. ν : **no unit**. Strain is a length divided by a length, so the units cancel.

(b) $210 \text{ GN/m}^2 = 210 \times 1000 = \mathbf{210\,000 \text{ N/mm}^2}$. $3.5 \text{ kNm} = 3500 \text{ N} \times 1000 \text{ mm} = \mathbf{3.5 \times 10^6 \text{ N mm}}$.

T2 Principle: axial load, so direct stress; Poisson's ratio for the diameter.

$A = \pi(40)^2/4 = 1256.6 \text{ mm}^2$; $\sigma = 120\,000/1256.6 = \mathbf{95.5 \text{ N/mm}^2}$ (compression).

$\varepsilon = 95.49/210\,000 = 4.547 \times 10^{-4}$; shortening $= 4.547 \times 10^{-4} \times 800 = \mathbf{0.364 \text{ mm}}$.

$\delta d = \nu \varepsilon d = 0.28 \times 4.547 \times 10^{-4} \times 40 = \mathbf{0.00509 \text{ mm}}$, an **increase**: a squashed bar gets fatter.

T3 Principle: find the longitudinal strain from the load, then use the definition of ν .

$A = \pi(25)^2/4 = 490.9 \text{ mm}^2$; $\varepsilon = \frac{60\,000}{490.9 \times 200\,000} = 6.112 \times 10^{-4}$.

Lateral strain $= 0.0046/25 = 1.84 \times 10^{-4}$; $\nu = 1.84 \times 10^{-4}/6.112 \times 10^{-4} = \mathbf{0.301}$.

Believable: yes. Steel is usually about 0.3, and ν must be below 0.5.

T4 Principle: strength pair $T/J = \tau/R$, with R the *outside* radius; then $P = 2\pi NT/60$.

$J = \pi(80^4 - 50^4)/32 = 3.408 \times 10^6 \text{ mm}^4$; $T = \tau J/R = 50 \times 3.408 \times 10^6/40 = 4.260 \times 10^6 \text{ N mm} = \mathbf{4.26 \text{ kNm}}$.

$P = 2\pi \times 400 \times 4260/60 = 178\,400 \text{ W} = \mathbf{178 \text{ kW}}$.

Check: the stress at the bore, $50 \times 25/40 = 31.3 \text{ N/mm}^2$, is lower than at the surface, as the straight-line stress profile predicts.

T5 At the same τ and R , torque is proportional to J ; weight is proportional to the area of the section.

(a) $\frac{T_h}{T_s} = \frac{60^4 - 40^4}{60^4} = 1 - \left(\frac{40}{60}\right)^4 = 1 - 0.1975 = \mathbf{0.802}$.

(b) $\frac{W_h}{W_s} = \frac{60^2 - 40^2}{60^2} = 1 - 0.4444 = \mathbf{0.556}$.

(c) Torque per kg, hollow $\div$ solid $= 0.802/0.556 = \mathbf{1.44}$: the hollow shaft carries about **44% more torque per kilogram**. It loses only 20% of the strength but 44% of the weight, because the material removed was near the centre, where the stress is low.

Quick reference

UNITS

Stress	$1 \text{ N/mm}^2 = 1 \text{ MPa} = 1 \text{ MN/m}^2$	$1 \text{ GN/m}^2 = 1000 \text{ N/mm}^2$
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DIRECT STRESS AND STRAIN

Stress	$\sigma = P/A, A = \pi d^2/4$	tension or compression
Strain	$\epsilon = \delta L/L$	no unit
Modulus	$E = \sigma/\epsilon \Rightarrow \delta L = PL/AE$	steel $\approx 200\,000 \text{ N/mm}^2$
Poisson	$\nu = (\delta d/d)/(\delta L/L), \delta d = \nu \epsilon d$	$0 < \nu < 0.5$; opposite sense

TORSION

Equation	$T/J = \tau/r = G\theta/L$	θ in radians
Power	$P = 2\pi NT/60$	N in rev/min, T in N m
Solid	$J = \pi d^4/32, \tau_{\max} = 16T/\pi d^3$	
Hollow	$J = \pi(D^4 - d^4)/32$	use $R = D/2$ for $\tau_{\max}$
Design	d from strength and from stiffness	take the larger

The five-question method

1 Given (in N and mm) • **2 Find** • **3 Principle**, and why • **4 Solve**: rearrange, then substitute • **5 Check**: unit, size, sense, second route.

Your next step

WHAT COMES NEXT

You can now read the stress, strain and torsion formulas, choose between them, rearrange them, and design a bar or a shaft. That is the foundation for the rest of Strength of Materials. **Still ahead** in the Module II Strength of Materials papers: shear force and bending moment diagrams, bending stress in beams, beam deflection, thin cylinders, strain energy and Castigliano's theorem, and springs. The papers also ask you to *derive* the torsion equation and spring formulas from first principles, which this starter does not do.

Ready to go further? The complete **SomaReady Strength of Materials Revision Guide** (36 pages) covers all eight outline topics in the same step-by-step style: every derivation the papers ask for, 41 original examples, and worked answers to every practice question. **Price: KSh 249** (launch price). **Get it at:** [LINK – to be confirmed].

About this guide. All explanations, examples, questions and diagrams are original SomaReady material. Every numerical answer was checked by an independent calculation. Topic counts come from 12 Module II papers and describe that sample only. SomaReady is an independent educational publisher; this guide is not produced, approved or endorsed by the Kenya National Examinations Council and does not guarantee any examination result. You may share this free guide unchanged. © 2026 SomaReady. SOM Formula-to-Solution Starter, Free Edition 1.